Fermi National Accelerator Laboratory

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Underestimation of Oxygen Deficiency Hazard through Use of Linearized Temperature Profiles

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Introduction

The failure mode analysis for any cryogenic system includes the effects of a large liquid spill due to vessel rupture or overfilling. The Oxygen Deficiency Hazard (ODH) analysis for this event is a strong function of the estimated heat flux entering the spilled liquid. A common method for estimating the heat flux is to treat the surface on which the liquid spills as a semi-infinite solid. This note addresses the effect of linearizing the temperature profile in this form of analysis, and shows it to cause the calculated flux to be underestimated by more than a factor of two.

System

The idealized system under consideration consists of a solid with one known surface, infinite in all other directions, of constant properties and at a uniform initial temperature, T_i (Fig. 1). When the spill occurs (t=0), the surface of the solid is brought to a temperature, T_b, equal to the boiling point of the spilled cryogen. The energy equation for the solid then reduces to the one-dimensional, transient conduction form

$$\frac{\delta^2 T(x,t)}{\delta x^2} = \frac{1}{\alpha}\frac{\delta T(x,t)}{\delta t} \qquad [1]$$

where the thermal diffusivity, α, is constant and the temperature, T, is a function of both position and time. The applied boundary conditions are

$$T(x,0) = T_i$$
$$T(0,t) = T_b$$

After solving for the temperature profile, the vaporizing heat flux can be calculated by applying Fourier's Law at the surface

$$q''(t)\Big|_{x=o} = -k\,\frac{\delta T(x,t)}{\delta x}\Big|_{x=o} \qquad [2]$$

From [2], the surface heat flux is seen to be directly related to the temperature gradient at the surface. This gradient has been evaluated by the approximate penetration depth method, and by using the closed form solution to [1], with the results compared.

Penetration Depth Method

The penetration depth method estimates the temperature profile in the solid by calculating the distance at which the effect of the surface condition is just beginning to be seen. A linear profile can then be constructed between this point and the surface, since the temperatures at each position are known (Fig. 2). Using notation from McAdams[1], the penetration depth, x_p, can be determined by equating the dimensionless term

$$z = \frac{x_p}{2\sqrt{\alpha t}} = 2 \qquad [3]$$

Rearranging, x_p is then

$$x_p = 4\sqrt{\alpha t} \qquad [4]$$

The known temperatures for the linear curve fit are

$$T(0,t) = T_b$$
$$T(x_p,t) = T_i$$

Applying Fourier's Law, the calculated heat flux vaporizing the liquid is

$$q''_{pd}(t)\Big|_{x=o} = -k\frac{(T_i-T_b)}{x_p} = -k\,\frac{(T_i-T_b)}{4\sqrt{\alpha t}} \qquad [5]$$

Closed Form Solution

An exact solution of the energy equation [1] does exist, and the temperature profile in the solid is given by

$$\frac{T(x,t)-T_b}{T_i-T_b} = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \qquad [6]$$

Differentiating [6] and setting x equal to 0 to evaluate the surface condition, the vaporizing heat flux is given by[2,3]

$$q''_{cf}(t)\Big|_{x=o} = -k\,\frac{(T_i - T_b)}{\sqrt{\pi \alpha t}} \qquad [7]$$

Results

A comparison of the heat flux to the liquid calculated using the penetration depth method, [5], and the closed form solution, [7], shows that:

$$\frac{q''_{pd}(t)}{q''_{cf}(t)} = \left[\frac{-k(T_i - T_b)}{4\sqrt{\alpha t}}\right]\left[\frac{\sqrt{\pi \alpha t}}{-k(T_i - T_b)}\right] = \frac{\sqrt{\pi}}{4} \qquad [8]$$
$$= 0.443$$

The use of a linear temperature profile approximation causes the vaporizing heat flux to be underpredicted by more than a factor of two.

Conclusions

For a semi-infinite solid, the calculated heat flux to the solid surface is lower by a factor of 2.26 if the temperature profile is linearized between the surface and the depth to which the solid has just begun to be affected by the surface condition. Other linear approximations would also underpredict the surface heat flux, by varying degrees. Since the exact solution is known, it should be used if the semi-infinite solid model is used to predict the vaporizing heat flux to the liquid from the ground.

In an ODH analysis, the use of a linear temperature profile could severely underpredict the vaporizing heat flux, the liquid boil-off rate, and the resulting asphyxiation hazard.

References

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3. Eckert, E.R.G., and Drake, R.M. Jr., Analysis of Heat and Mass Transfer, 1st edition, McGraw Hill, New York, 1972, p.160–164.

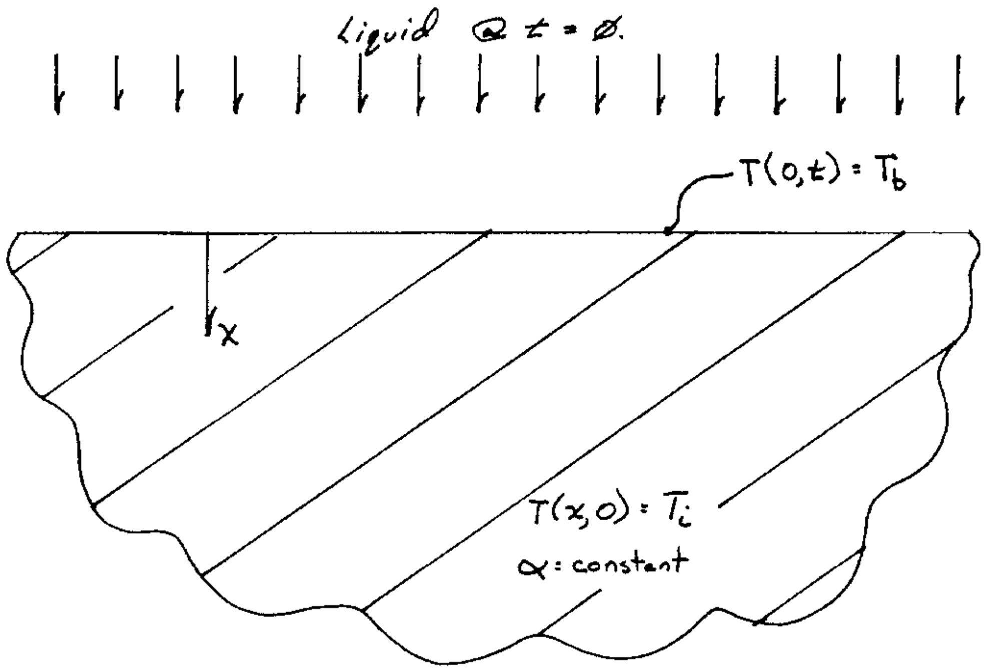

Figure 1. Semi-infinite Solid Model

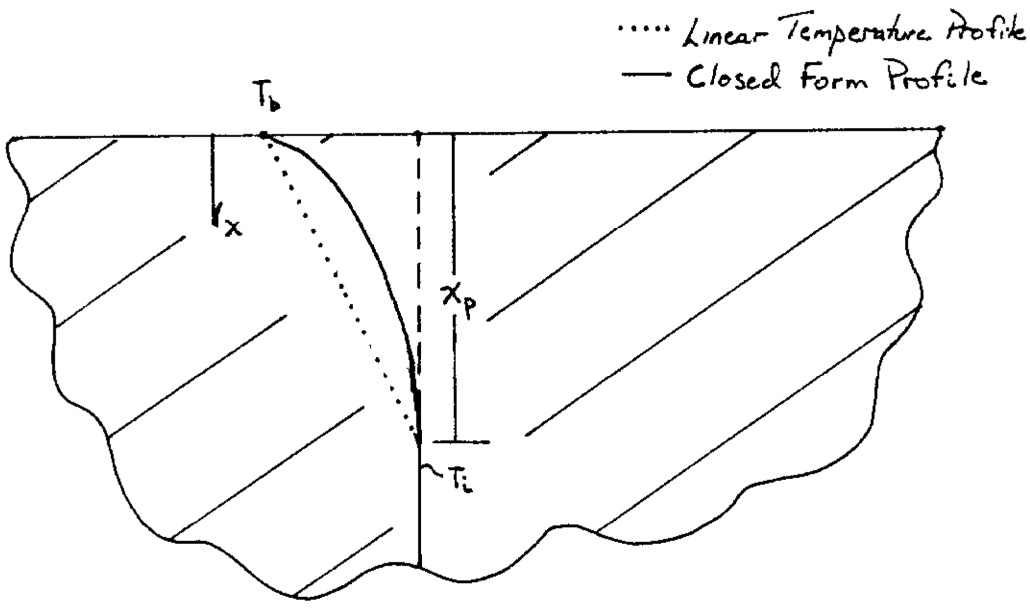

Figure 2. Penetration Depth and Closed Form Temperature Profiles